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DIRECT AND INVERSE THEOREMS ON STATISTICAL APPROXIMATIONS BY POSITIVE LINEAR OPERATORS

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ABSTRACT. In the present paper, we obtain some direct and inverse results on A-statistical convergence of the sequence of a general positive linear operators.

We also compute the order of A-statistical convergence by means of modulus of continuity and Peetre's K-functionals.

1. Introduction. Before proceeding further we recall some notations on the concept of statistical and A-statistical convergence.

The concept of statistical convergence was introduced by Fast [**12**]. A sequence $x = (x_k)$ is said to be statistically convergent to a number L if, for every $\varepsilon > 0$, $\delta\{k \in \mathbf{N} : |x_k - L| \geq \varepsilon\} = 0$, where $\delta(K)$ is the natural density of the set $K \subseteq \mathbf{N}$. Recall that the subset $K \subseteq \mathbf{N}$ has density [**22**] if

$$\delta(K) := \lim_n \frac{1}{n}\{\text{the number } k \leq n : k \in K\}$$

exists. Notice that, any convergent sequence is statistically convergent but not conversely. For instance, the following sequence

$$x_k = \begin{cases} L_1 & \text{if } k = m^2, (m = 1, 2, 3, \dots) \\ L_2 & \text{if } k \neq m^2 \end{cases}$$

is statistically convergent to L_2 but not convergent in ordinary sense when $L_1 \neq L_2$.

Let $A = (a_{nk})$, $k, n = 1, 2, \dots$, be an infinite summability matrix. The A-transform of the sequence $x = (x_k)$, denoted by $Ax := ((Ax)_n)$,

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