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GLOBAL DYNAMICS IN A TB MODEL INCORPORATING CASE DETECTION AND TWO TREATMENT STAGES

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ABSTRACT. Case detection of an infectious individual and differentiation of infectiveness of a treated patient during two different stages of treatment are recognized as among key factors for the successful control and management of tuberculosis (TB) transmission. In this paper, a dynamic compartmental model is developed that incorporates these factors, and proofs are provided to show that the model's global dynamics are completely characterized by the control reproduction number, and in particular the disease eradication condition in terms of the case detection fraction is obtained, along with some numerical simulations.

1. Introduction. Tuberculosis (TB) caused by infection with the bacterium M. tuberculosis is an ancient and chronic infectious disease. It is estimated that one-third of the world's population has been infected with M. tuberculosis, resulting in nearly 3 million deaths each year [**2, 3, 18**]. Furthermore, there are more than 6.5 million new cases of tuberculosis each year [**20**].

Many mathematical models have been proposed and analyzed to examine TB transmission dynamics, and to suggest and evaluate control strategies [**4, 5, 6, 8, 12**]. In particular, issues such as vaccination, drug-resistance, the reinfection and relapse of cured individuals have been addressed in different models [**7, 10, 17**]. Of particular concern in this paper is the impact of case detection on an effective treatment program. This is motivated by the observation that, in China, a fraction of case detection of smear-positive pulmonary tuberculosis was only 41.4

Keywords and phrases. TB model, case detection, relapse, treatment, M. tuberculosis.

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