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INTEGER SOLUTIONS TO THE EQUATION $y^2 = x(x^2 \pm p^k)$

P.G. WALSH

1. Introduction. Let p be a prime number and $k \geq 1$ an integer. In a recent paper [**8**], Draziotis determines the integer solutions (x, y), with $y > 0$, to the diophantine equations

$$y^2 = x(x + p^k)(x - p^k), \tag{1.1}$$

$$y^2 = x(x^2 - p^k), \quad (k \text{ odd}) \tag{1.2}$$

and

$$y^2 = x(x^2 + p^k). \tag{1.3}$$

Note that if (x, y) is a solution to any one of these equations, then (p^2x, p^3y) is a solution to the same equation, but with k replaced by $k+4$. This remark motivates the notion of a primitive integer solution to the above equations.

Definition. An integer solution (x, y) to a Diophantine equation of the form $y^2 = x^3 \pm p^k x$ is *primitive* if $y > 0$ and p^2 does not divide x. If p^2 divides x, then (x, y) is referred to as an *imprimitive* solution.

In order to determine all of the integer solutions to the equations above, it is sufficient to determine the primitive and imprimitive integer solutions. In [**8**], there are a number of shortcomings in the statements of the results that we endeavor to clarify and sharpen. In particular, no consideration to the concept of primitive solutions is given, thereby resulting in an algorithm which has endless running time (as k goes to infinity), and repeatedly finds imprimitive points of the form $(p^{2t}x, p^{3t}y)$ for some positive integer t. Furthermore, the description of the integer solutions to (1.1)–(1.3) in [**8**] is given in terms of the solutions (a, b) to the equation $a^4 \pm b^2 = p^k$, which seems to simply be

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