ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 4, 2008

A NESTED EMBEDDING THEOREM FOR HARDY-LORENTZ SPACES WITH APPLICATIONS TO COEFFICIENT MULTIPLIER PROBLEMS

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ABSTRACT. We prove a nested embedding theorem for Hardy-Lorentz spaces and use it to find coefficient multiplier spaces of certain nonlocally convex Hardy-Lorentz spaces into various target spaces such as Lebesgue sequence spaces, other Hardy spaces and analytic mixed norm spaces.

1. Introduction. In this paper we characterize coefficient multipliers between certain types of analytic function spaces on the open unit disk. We are primarily concerned with multipliers having one of the nonlocally convex Hardy-Lorentz spaces $H^{p,q}$, $0 < p < 1$, $0 < q \leq \infty$, for the domain space. For such multipliers we will consider a variety of target spaces including Lebesgue sequence spaces, other Hardy spaces and various analytic function spaces of mixed norm type. Our method depends upon a nested embedding theorem for Hardy-Lorentz spaces (Theorem 4.1) obtained through interpolation from embedding theorems of Hardy and Littlewood and of Flett. Thus, the strategy is to trap $H^{p,q}$ between a pair of mixed norm spaces of Bergman-type and then deduce multiplier results for $H^{p,q}$ from corresponding known multiplier results for the endpoint spaces.

The paper is organized as follows. In Section 2 we define the Hardy-Lorentz spaces and the analytic mixed norm spaces. Also included in this section are some results from interpolation, fractional calculus and H^p- theory needed for the sequel. Our primary references for Lorentz spaces and Hardy spaces are [**5, 10**], respectively. Section 3 covers preliminary material on coefficient multipliers. In Section 4 we state and prove the embedding theorem for $H^{p,q}$. We then indicate how this theorem may be used to obtain the duality results of [**33**]. In

2000 AMS *Mathematics subject classification.* Primary 30D55, 30H05, 42A16, 46A16, 46A45, 46E10, 46E15, 46E30.
Keywords and phrases. Hardy-Lorentz, nonlocally convex, dual space, coefficient, multiplier, mixed norm, Bergman.
Received by the editors on June 28, 2005, and in revised form on March 28, 2006.
DOI:10.1216/RMJ-2008-38-4-1215