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APPLYING THE CONLEY INDEX TO FAST-SLOW SYSTEMS WITH ONE SLOW VARIABLE AND AN ATTRACTOR

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ABSTRACT. Chay and Keizer [**3**] created a five-dimensional model of bursting activity in pancreatic β-cells which was subsequently reduced to a three-dimensional model by Chay [**2**]. Kinney has used the Conley index to show that the three-dimensional model has a nonempty attractor [**11**, pages 451–472]. This paper is intended to provide an introduction to the Conley index by showing how it can be applied to extend these results to prove the existence of a periodic orbit for the three-dimensional model, the existence of a nonempty attractor for the five-dimensional model and the existence of a periodic orbit for the five-dimensional model.

1. Introduction. Conley index theory consists of topological and algebraic tools for understanding the global dynamics of flows and maps on compact invariant sets. It is useful for proving the existence of various objects and properties, such as equilibria, periodic orbits [**15**], connecting orbits [**7, 8, 13, 16, 18, 23, 24, 26, 30, 34**], traveling waves [**7, 16, 18, 23, 30, 34**] and chaotic dynamics [**19, 20**]. Some fundamental references for the theory include [**6**], which traces its early development in the context of smooth flows on manifolds, Conley's classic monograph [**4**], Salamon's paper [**31**] for clarity of proof, Rybakowski and Smoller's books [**30, 34**] for applications to partial differential equations, and Mischaikow and Mrozek's surveys [**17, 21**]. Also recommended is an article by Moeckel [**25**] for an intuitive introduction to the index and to the related topic of connection matrices. The purpose of this paper is to introduce the reader to the Conley index by showing how even basic aspects of the theory can be used to prove nontrivial results in the context of certain kinds of fast-slow singular perturbation problems. More general references for

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