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SPECTRAL THEORY FOR NONLOCAL DISPERSAL WITH PERIODIC OR ALMOST-PERIODIC TIME DEPENDENCE

V. HUTSON, W. SHEN AND G.T. VICKERS

ABSTRACT. In applications to spatial structure in biology and to the theory of phase transition, it has proved useful to generalize the idea of diffusion to a nonlocal dispersal with an integral operator replacing the Laplacian. We study the spectral problem for the linear scalar equation

$$u_t(x,t) = \int_\Omega K(x,y)u(y,t)\,dy + h(x,t)u(x,t),$$

and tackle the extra technical difficulties arising because of the lack of compactness for the evolution operator defined by the dispersal. Our aim is firstly to investigate the extent to which the idea of a periodic parabolic principal eigenvalue may be generalized. Secondly, we obtain a lower bound for this in terms of the corresponding averaged spatial problem, and then extend this to the principal Lyapunov exponent in the almost periodic case.

1. Introduction. Recently there has been extensive investigation into a class of models for nonlocal spatial dispersal, in which the dispersal operator D, say, involves an integral operator, for example

$$(Du)(x) = \int_\Omega K(x,y)[u(y) - u(x)]\,dy. \tag{1.1}$$

Such models occur in a number of applications, for example biology and the theory of phase transition, as a generalization of classical diffusion where $D = \Delta$, the Laplacian with a suitable boundary condition. The derivation in the biological context is discussed in [**12, 17, 22**], and for the theory of phase transition, see [**5 , 7, 8**]. The nonlinear theory has

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