ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 4, 2008

ATTRACTORS FOR SEMI-LINEAR EQUATIONS OF VISCOELASTICITY WITH VERY LOW DISSIPATION

S. GATTI, A. MIRANVILLE, V. PATA AND S. ZELIK

ABSTRACT. We analyze a differential system arising in the theory of isothermal viscoelasticity. This system is equivalent to an integrodifferential equation of hyperbolic type with a cubic nonlinearity, where the dissipation mechanism is contained only in the convolution integral, accounting for the past history of the displacement. In particular, we consider here a convolution kernel which entails an extremely weak dissipation. In spite of that, we show that the related dynamical system possesses a global attractor of optimal regularity.

1. Introduction. Let $\Omega \subset \mathbf{R}^3$ be a bounded domain with smooth boundary $\partial\Omega$. For $t \in \mathbf{R}^+ = (0, \infty)$, we consider the evolution system arising in the theory of isothermal viscoelasticity [**9, 20**]

$$\begin{cases} \partial_{tt} u - \Delta u - \int_0^\infty \mu(s) \Delta \eta(s)\, ds + g(u) = f, \\ \partial_t \eta = T\eta + \partial_t u, \end{cases} \tag{1.1}$$

where $u = u(t) : \Omega \times [0, \infty) \to \mathbf{R}$, $\eta = \eta^t(s) : \Omega \times [0, \infty) \times \mathbf{R}^+ \to \mathbf{R}$ and $T = -\partial_s$, supplemented with the boundary and initial conditions

$$\begin{cases} u(t)|_{\partial\Omega} = \eta^t|_{\partial\Omega} = \eta^t(0) = 0, \\ u(0) = u_0, \quad \partial_t u(0) = v_0, \quad \eta^0(s) = \eta_0(s). \end{cases} \tag{1.2}$$

Here, $g : \mathbf{R} \to \mathbf{R}$ is a nonlinear term of (at most) cubic growth satisfying some dissipativity conditions, $f : \Omega \to \mathbf{R}$ is an external force, whereas the memory kernel μ is an absolutely continuous summable decreasing, thus nonnegative, function defined on $\mathbf{R}^+$. Problem (1.1)–(1.2) is cast

2000 AMS *Mathematics subject classification.* Primary 35B40, 35L70, 37L45, 45K05, 74D99.
Keywords and phrases. Hyperbolic equation with memory, dynamical system, Lyapunov function, gradient system, global attractor.
The fourth author has been supported by *CRDF* grant RUM1-2654-MO-05.
Received by the editors on December 22, 2005, and in revised form on April 4, 2006.
DOI:10.1216/RMJ-2008-38-4-1117