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CONFLUENT MAPPINGS AND ARC KELLEY CONTINUA

J.J. CHARATONIK†, W.J. CHARATONIK AND J.R. PRAJS

ABSTRACT. A Kelley continuum X, also called a continuum with the property of Kelley, such that, for each $p \in X$, each subcontinuum K containing p is approximated by arcwise connected continua containing p, is called an *arc Kelley continuum*. A continuum homeomorphic to the inverse limit of locally connected continua with confluent bonding maps is said to be *confluently* $\mathcal{LC}$*-representable*. The main subject of the paper is a study of deep connections between the arc Kelley continua and confluent mappings. It is shown that if a continuum X admits, for each $\varepsilon > 0$, a confluent ε-mapping onto a(n) (arc) Kelley continuum, then X itself is a(n) (arc) Kelley continuum. In particular each confluently $\mathcal{LC}$-representable continuum is arc Kelley. It is also proved that if continua X and Y are confluently $\mathcal{LC}$-representable, then also are their product $X \times Y$ and the hyperspaces 2^X and $C(X)$.

1. Introduction. The arc Kelley continua form a natural subclass of Kelley continua, known also as continua with the property of Kelley, or continua with property κ. In a recent study the authors proved that each absolute retract for any of the classes of: hereditarily unicoherent continua, tree-like continua, λ-dendroids and dendroids, is an arc Kelley continuum, [**9**]. All absolute retracts mentioned above share this property with all members of another significant class, the class $\mathcal{LC}$ of locally connected continua. One of the basic and essential results of the previous study says that every confluently $\mathcal{T}$-representable continuum (i.e., the inverse limit of trees with confluent bonding mappings, $\mathcal{T}$ stands for the class of trees) is an absolute retract for hereditarily unicoherent (tree-like) continua, [**11**]. These results led to the question whether there is some deep connection between confluent mappings and arc Kelley continua. For instance, is the inverse limit of arc

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