

## XIA'S ANALYTIC MODEL OF A SUBNORMAL OPERATOR AND ITS APPLICATIONS

JIM GLEASON AND C. RAY ROSENTRATER

**ABSTRACT.** In this paper, we provide an exposition and some applications of Xia's analytic model for pure subnormal operators and the associated mosaic. Using the model, we provide a complete set of examples of subnormal operators with rank one or two self-commutator.

**1. Preface.** One of the best ways to prove theorems about operators contained in a certain category is to show that all operators in the category can be expressed in a certain way. Examples of this include the model of multiplication by the independent variable on an  $L^2(\mu)$  space for  $*$ -cyclic normal operators and the Toeplitz type model for hyponormal operators of Sz.-Nagy and Foiaş. In the late 1980's, Daoxing Xia created such a model for subnormal operators. In his papers, Xia proves that every pure subnormal operator can be expressed as multiplication by the independent variable on a certain type of vector-valued  $R^2$  space.

The purpose of this paper is to give an exposition of this work. We offer new proofs of some of the results and provide many examples to help clarify the model. We also give several applications that have resulted from using the Xia model. It is our hope that this exposition encourages more work in the area. To that end we conclude each section with "Notes and open problems" which gives some additional background to the topics covered in the section as well as some of the related open problems. Furthermore, for the sake of brevity, only illustrative proofs are included. For the complete set of proofs and computations, the reader should contact the authors.

We have tried to be thorough and include all of the results related to the Xia model and to make correct references to theorems that have previously been published. For any omissions, we apologize.

Section 2 gives a thorough exposition of the works of Xia concerning his analytic model. We also describe some of the complete unitary

---

Received by the editors on June 7, 2004.

DOI:10.1216/RMJ-2008-38-3-849 Copyright ©2008 Rocky Mountain Mathematics Consortium