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SEPARABILITY AND EFFICIENCY UNDER STANDARD WREATH PRODUCT IN TERMS OF CAYLEY GRAPHS

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ABSTRACT. In this paper we are mainly interested in separability and efficiency under the standard wreath product. To do that we will first obtain a presentation, say $\mathcal{P}_G$, for the standard wreath product in terms of Cayley graphs. Then we will prove our first main result of this paper, which can be thought of as an application of the result given in [**17**] (or the general result in [**6**]). Moreover, by considering the standard wreath product G of any finite groups B by A, we will define the relationship between B-separability and efficiency, on G, as another main result of this paper.

1. Introduction. Let G be a group, and let H be a subgroup of G. Then G is said to be *H-separable* if, for each $x \in G - H$, there exists $N \lhd G$ with finite index such that $x \notin NH$. Moreover, G is called *subgroup separable* if G is H-separable for all finitely generated subgroups H of G. The best known results about subgroup separability can be found, for instance, in [**2**, Section 3], [**11, 14**]. Furthermore, let S be a generating set for G. The *Cayley graph*, see, for example, [**3, 8, 12, 13**], of G, denoted by Γ_G, with respect to S has a vertex for every element of G, with an edge g to gs for all elements $g \in G$ and $s \in S$. Thus, the initial vertex of the edge is g and the terminal is gs.

Suppose that G is the *semi-direct product* of a group K by a group A, denoted by $K \rtimes_\theta A$ where $\theta : A \to \mathrm{Aut}\,(K)$, $a \to \theta_a$, $a \in A$, is a homomorphism. Suppose also that $\mathcal{P}_K = \langle \underset{=}{y}; \underset{=}{s} \rangle$ and $\mathcal{P}_A = \langle \underset{=}{x}; \underset{=}{r} \rangle$ are the presentations for the groups K and A, respectively, under the maps $y \to k_y$, $y \in \underset{=}{y}$, $x \to a_x$, $x \in \underset{=}{x}$. (We note that, throughout this paper, the notation "$\underset{=}{\cdot}$" used in group presentations denotes the finite set of

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