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SOME NEAR-RINGS IN WHICH ALL IDEALS ARE INTERSECTIONS OF NOETHERIAN QUOTIENTS

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ABSTRACT. For every near-ring, Noetherian quotients are one source of ideals, but usually not all ideals can be obtained from such quotients. In this paper, we show that every ideal of a zero symmetric ring-free tame near-ring with identity is dense in the intersection of the Noetherian quotients that contain it. In many cases, we are able to determine the ideal lattice of the near-ring of those functions on a group that are compatible with a given subset of the set of all normal subgroups. In particular, let G be a finite group, and let $\{0\} = A_1 < A_2 < \cdots < A_{n-1} < A_n = G$ be a chain of normal subgroups of G with $|A_i/A_{i-1}| \geq 3$ for all $i \in \{2, \ldots, n\}$. Then the lattice of ideals of the near-ring of zero-preserving functions compatible with A_i for all i is shown to consist entirely of intersections of Noetherian quotients. The unique minimal ideal of these near-rings is explicitly determined.

1. Motivation. We will compute the ideal lattice of certain finite near-rings. For most of the well-studied function near-rings, such as the inner automorphism near-ring on a given finite group, the lattice of ideals is not known. However, using *Noetherian quotients* [**10**, Definition 1.41], one obtains many ideals of a given function near-ring, and often all maximal ideals [**1**, Theorem 1.2]. In [**11**, Problem 5], Scott proposed the problem to find all ideals for a certain type of function near-rings. He conjectured that all ideals of these near-rings could be found as intersections of Noetherian quotients. In [**3, 4**] this problem was solved. In most cases, all ideals were in fact intersections of Noetherian quotients, and in one case, one additional ideal appeared. In this paper, we will exhibit a large class of near-rings in which all ideals are intersections of Noetherian quotients.

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