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ORLICZ SPACES OF INTEGRABLE FUNCTIONS WITH RESPECT TO VECTOR-VALUED MEASURES

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ABSTRACT. This paper extend the theory of p-integrable functions with respect to a vector measure studied in [**6**] for $p = 1$ and in [**10**] for $1 < p < \infty$, introducing the same notion in the context of the Orlicz spaces. Some topics of the third section have been treated recently in [**1**] but with a different point of view.

1. Introduction. After the appearance of Orlicz spaces, the theory of L_p-spaces has progressed in several directions. The special structure of Orlicz spaces discovers an amount of new questions that are hidden in the classical theory because of the special properties of L_p-spaces. Moreover, Orlicz spaces can be the first step to consider more abstracts generalizations to the classical theory of Banach spaces. The vigorous growth of the topic is a consequence of the interest of applications to potential theory, interpolation and differential equations among others.

As in classical theory of scalar integration, the spaces of integrable functions with respect to a vector measure has been constructed around L_p-spaces. This paper tries to introduce the integration with respect to vector-valued measures in the setting of Orlicz spaces. As we can see, this extension carries new problems, the solution of which is here not as clear as in classical theory.

In the following, (Ω, Σ, μ) denotes a measure space, where Σ is a σ-algebra and μ a nonnegative measure. For definitions and properties of Orlicz functions and the Orlicz spaces of measurable functions we refer to [**3, 7, 9**]. A nondegenerated Orlicz function Φ is a continuous, nondecreasing and convex function defined in $[0, \infty[$ such that $\Phi(t) = 0$ if and only if $t = 0$ and $\lim_{t\to\infty} \Phi(t) = \infty$. The representation integral of a convex function Φ can be used to obtain interesting properties;

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