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ON MAXIMUM MODULUS POINTS AND ZERO SETS OF ENTIRE FUNCTIONS OF REGULAR GROWTH

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ABSTRACT. Let f be an entire function. We denote by $R(w,f)$ the distance between a maximum modulus point w and the zero set of f. In a previous paper, the authors obtained asymptotical lower bounds for $R(w,f)$ as $|w| \to \infty$ for functions of finite positive order and regular growth. In this work we extend those results to functions of either zero or infinite order and show that our results are sharp in sense of order.

1. Introduction. Let f be an entire function. We call a point $w \in \mathbf{C}$ a *maximum modulus point* if

$$|f(w)| = M(|w|, f),$$

where

$$M(r,f) := \max_{|z|=r} |f(z)|.$$

We denote by $R(w,f)$ the distance between a maximum modulus point w and zero set of f, i.e.,

$$R(w,f) := \inf\{|w-z| : f(z) = 0\}.$$

Lower estimates of $R(w,f)$ play an important role in Macintyre's version [**7**] of the Wiman-Valiron theory and its further generalizations, see [**4**, Chapter 1, Section 4] and [**11**].

Theorem A [**7**]. (i) *The following inequality holds*

$$\limsup_{|w|\to\infty} \frac{R(w,f)}{|w|} (\log M(|w|,f))^{1/2} > 0.$$

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