ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 2, 2008

# ON A MODEL FOR PHASE TRANSITIONS WITH VECTOR HYSTERESIS EFFECT

EMIL MINCHEV

ABSTRACT. The paper deals with a system of nonlinear PDEs which describes a phase transition model with vector hysteresis and diffusion effect. Existence of solutions for the system under consideration is proved by the method of Yosida approximation, $L^\infty$-estimates and energy type inequalities in $L^2$.

**1. Introduction.** The present paper deals with a system of nonlinear PDEs which is a model of a class of phase transitions where the hysteresis and diffusive effects are taken into account:

$$a\mathbf{w}_t - \kappa\Delta\mathbf{w} + \partial\mathbf{I}_{K(u)}(\mathbf{w}) \ni \mathbf{F}(\mathbf{w}, u) \quad \text{in} \quad Q, \tag{1}$$

$$\mathbf{c}\cdot\mathbf{w}_t + du_t - \Delta u = h(\mathbf{w}, u) \quad \text{in} \quad Q. \tag{2}$$

Here $N$ and $m$ are positive integers, $\mathbf{w} = (w_1, \dots, w_m)$, $T > 0$, $\Omega \subset R^N$ is a bounded domain with smooth boundary $\partial\Omega$, $Q = (0,T)\times\Omega$; $a$, $\kappa$, $\mathbf{c} = (c_1, \dots, c_m)$, $d$ are given constants; $\mathbf{F} : R^m \times R \to R^m$, $h : R^m \times R \to R$, ${f_i}_*, {f_i}^* : R \to R$, $i = 1, \dots, m$, are given functions. We assume that ${f_i}_*, {f_i}^* \in C^2(R)$, ${f_i}_* \le {f_i}^*$ on $R$ and there exist constants $k_i > 0$ such that ${f_i}_* = {f_i}^*$ on $(-\infty, -k_i] \cup [k_i, \infty)$, $i = 1, \dots, m$.

For each $u \in R$ we denote by $\partial I_u^{(i)}(\cdot)$ the subdifferential of the indicator function $I_u^{(i)}(\cdot)$ of the interval $[f_{i*}(u), f_i^*(u)]$, $i = 1, \dots, m$, namely,

$$I_u^{(i)}(w_i) = \begin{cases} 0 & \text{if } f_{i*}(u) \le w_i \le f_i^*(u) \\ +\infty & \text{otherwise} \end{cases}$$

---

2000 AMS *Mathematics subject classification.* Primary 35R70, 35K50, 74N30.
*Keywords and phrases.* Nonlinear PDEs, existence of solutions, subdifferential, Yosida approximation, hysteresis, $L^\infty$-energy method.
Received by the editors on November 11, 2004, and in revised form on January 16, 2006.
DOI:10.1216/RMJ-2008-38-2-545 