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STRUCTURALLY STABLE QUADRATIC FOLIATIONS

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ABSTRACT. We characterize the elements of F_n, the set of polynomial vector fields on the plane of degree at most n without finite singular points, that are structurally stable with respect to perturbations within F_n for $n \leq 2$. We do so with respect to each of the two natural definitions of stability in this setting.

1. Introduction and statement of the main results. Although the characterization of C^r vector fields on compact two-dimensional manifolds that are structurally stable goes back to Peixoto [**11**], the characterization of all structurally stable planar polynomial vector fields of degree n (under perturbation by polynomial vector fields of degree at most n) is still an open problem since it is not known if the "natural" hyperbolicity condition on the limit cycles is needed for stability.

Moreover, even in the case of general families for which we know a *characterization theorem* in terms of singular points, periodic orbits and saddle connections, it is difficult to give an explicit *classification* of all structurally stable phase portraits. An exception to this can be found in [**2**], in which is given the classification of all structurally stable phase portraits for the quadratic family modulo limit cycles. We note, however, that the mathematical object that is structurally stable is not the phase portrait but the specific vector field that realizes it.

One family of polynomial vector fields that is a natural candidate for a complete characterization theorem, and a categorization of structurally stable phase portraits, is what we will refer to as the set of *planar polynomial foliations*, or simply, *foliations*, that is, planar polynomial

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