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SMOOTHNESS PROPERTIES OF QUASI-MEASURES

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ABSTRACT. We construct several examples of simple quasi-measures that show that the strong and weak smoothness properties for quasi-measures proposed by Boardman are distinct. These examples also show that in general, only the obvious implications hold between these properties. We describe a general construction of product quasi-measures that yields further examples that are not simple.

We also provide characterizations of the strong smoothness properties in terms of the action of the induced Borel quasi-measure on the Stone-Čech compactification and show that the dimension of the Stone-Čech remainder influences the smoothness properties of the Baire quasi-measures on X. Finally, we explore the effects of different topological properties on the various classes of smooth quasi-measures.

1. Introduction. Quasi-measures were first studied by Johan Aarnes [**1**] on compact spaces as set functions that represent functionals which are linear on singly generated subalgebras of the collection of real-valued continuous functions. They are generalizations of the regular measures which appear in the Riesz representation theorem. Later, Boardman [**5, 6**], generalized Aarnes' results to the case where the underlying space is completely regular and asked several questions relating to the smoothness properties of quasi-measures in this context. The goal of this paper is to answer those questions. Further information about quasi-measures in this setting can be found in [**4**] where the representation theorem of Boardman is proved in a cleaner way and in [**9**] where some topological properties of the collection of quasi-measures are addressed.

All spaces X under consideration are assumed to be completely regular. A *Baire quasi-measure* on X is a real-valued, finite, non-negative set function μ defined on $\mathcal{A} = \{A \subseteq X : A \text{ is either a zero set or cozero set of } X\}$ that satisfies the following axioms:

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