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FOURTH ORDER OPERATORS WITH GENERAL WENTZELL BOUNDARY CONDITIONS

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ABSTRACT. Let Ω be a bounded subset of $\mathbf{R}^N$ with smooth boundary $\partial\Omega$ in C^4, $a \in C^4(\overline{\Omega})$ with $a > 0$ in $\overline{\Omega}$, and let A be the fourth order operator defined by $Au := \Delta(a\Delta u)$, respectively $Au := B^2 u$, where $Bu := \nabla \cdot (a\nabla u))$, with general Wentzell boundary condition of the type

$$Au + \beta \frac{\partial(a\Delta u)}{\partial n} + \gamma u = 0 \quad \text{on} \quad \partial\Omega,$$

$$\left(\text{respectively } Au + \beta \frac{\partial(Bu)}{\partial n} + \gamma u = 0 \quad \text{on} \quad \partial\Omega\right).$$

We prove that, under additional boundary conditions, if $\beta, \gamma \in C^{3+\varepsilon}(\partial\Omega)$, $\beta > 0$, then the realization of the operator A on a suitable Hilbert space of L^2 type, with a suitable weight on $\partial\Omega$, is essentially self-adjoint and bounded below.

0. Introduction. Consider problems involving the Laplacian Δ on a smooth bounded domain Ω in $\mathbf{R}^N$. The usual boundary conditions are of Robin type, i.e.,

$$\beta \frac{\partial u}{\partial n} + \gamma u = 0,$$

where $(\beta(x), \gamma(x))$ is a nonzero vector for each $x \in \partial\Omega$, the boundary of Ω, and n is the unit outer normal to $\partial\Omega$. But by working in $C(\overline{\Omega})$ rather than in $L^p(\Omega)$ one can use Wentzell boundary conditions of the form

$$\alpha\Delta u + \beta \frac{\partial u}{\partial n} + \gamma u = 0,$$

where $(\alpha(x), \beta(x), \gamma(x))$ is a nonzero vector in $\mathbf{R}^3$ for each x in $\partial\Omega$. The resolvent equation $\Delta u - \lambda u = h$ on the boundary cannot distinguish between $u = 0$ on $\partial\Omega$ and $\Delta u = 0$ on $\partial\Omega$ when $h = 0$ on $\partial\Omega$; such functions h are dense in $L^2(\Omega)$ but not in $C(\overline{\Omega})$. In the previous work

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