CONJUGACY CRITERIA FOR THE HALF-LINEAR SECOND ORDER DIFFERENTIAL EQUATION

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ABSTRACT. In this paper, some conjugacy criteria for the half-linear second order differential equation

$$\left(|y'(t)|^{p-1}\mathrm{sgn}\,y'(t)\right)' + c(t)|y(t)|^{p-1}\mathrm{sgn}\,y(t) = 0, \quad p > 1$$

are obtained.

1. Introduction. We are concerned with the zeros of solutions of the half-linear second order differential equation

$$(1.1) \qquad \left(\phi(y'(t))\right)' + c(t)\phi(y(t)) = 0$$

where $c(t)$ is a continuous function on $\mathbf{R}$ and $\phi(s)$ is the real function defined by $\phi(s) := |s|^{p-1}\mathrm{sgn}\,s$, with $p > 1$. In the case $p = 2$, equation (1.1) reduces to the linear equation

$$(1.2) \qquad y''(t) + c(t)y(t) = 0.$$

The investigation of qualitative properties of (1.1) was initiated by Elbert who proved that the zeros of linearly independent solutions of (1.1) interlace and that the Sturm comparison theorem extends to half-linear equations [**7**]. Moreover, he had attracted the attention of many authors for an expected similarity between the qualitative properties of (1.1) and (1.2).

As in the case of linear equations, equation (1.1) is called *disconjugate* if it has a solution without any zeros. Otherwise, it is called *conjugate*.

Concerning the linear case, conjugacy criteria for (1.2) and the general equation

$$(1.3) \qquad (a(t)y'(t))' + c(t)y(t) = 0$$