ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 1, 2008

THE CHARACTERIZATION OF MOORE-PENROSE INVERSE MODULE MAPS AND THEIR CONTINUITY

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ABSTRACT. In this paper we will introduce the concept of *Moore-Penrose inverse module map* and an equivalent characterization, provided that E and F are both (right) Hilbert C^*-modules over a C^*-algebra A, with $L(E,F)$ the set of all adjointable A-module maps from E to F. Then we use the concept as a main tool in obtaining a Douglas type factorization theorem about certain important bounded module maps. Thus, we come to the discussion of the continuity of Moore-Penrose inverse module maps depending upon a parameter: Let X be a topological space and $x \mapsto T_x : X \mapsto L(E)$ a continuous map, with $R(T_x)$ a closed submodule in E for each $x \in X$. Then the Moore-Penrose inverse module map T_x^+ of T_x is continuous if and only if $||T_x^+||$ is locally bounded. Furthermore, this is equivalent to the following statement:

For any x_0 in X, there exists a neighborhood U_0 of x_0 and a positive number λ such that $(0, \lambda^2) \subseteq \mathbf{C} \setminus \sigma(T_x^* T_x)$ for all $x \in U_0$, where $\sigma(T)$ denotes the spectrum of the operator T.

1. Introduction. Hilbert C^*-modules constitute a frequently used tool in operator theory and operator algebras. Research fields benefiting from it include K-theory, index theory for operator-valued conditional expectations, group representation theory, operator-valued free probability and investigations into compact quantum groups, generalized Atiyah-Singer index theorems and topological invariants. Besides these, the theory of Hilbert C^*-modules is very interesting in its own right.

It is well known that Moore-Penrose inverse matrices and Moore-Penrose inverse operators play an important role in matrix theory and in operator theory, respectively. Meanwhile, in the study of factorization of Hilbert C^*-module maps, there is no suitable tool to use. Motivated by the above observation, in this paper we will introduce the concept of *Moore-Penrose inverse module maps* and an

2000 AMS *Mathematics subject classification.* Primary 47L05, Secondary 46C50.
Received by the editors on May 8, 2005, and in revised form on October 5, 2005.
DOI:10.1216/RMJ-2008-38-1-351