ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 1, 2008

Q_K SPACES VIA HIGHER ORDER DERIVATIVES

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ABSTRACT. We characterize the Möbius invariant Q_K spaces in terms of higher order derivatives. Our methods are new even in the case of Q_p spaces.

1. Introduction. One of the classical topics in complex analysis is the study of Möbius invariant function spaces in the unit disk $\mathbf{D}$, namely, spaces of analytic functions equipped with a norm that is invariant under the action of Möbius maps. Examples of such spaces include the familiar disk algebra, H^∞, the Dirichlet space, the Bloch space and BMOA.

A particular class of Möbius invariant function spaces, the so-called Q_p spaces, has attracted a lot of attention in recent years. More specifically, for any $0 \le p < \infty$, Q_p consists of all analytic functions f in the unit disk $\mathbf{D}$ such that

$$\|f\|_{Q_p}^2 = \sup_\varphi \int_{\mathbf{D}} |f'(z)|^2 (1 - |\varphi(z)|^2)^p \, dA(z) < \infty,$$

where dA is area measure on $\mathbf{D}$ normalized so that $A(\mathbf{D}) = 1$, and the supremum is taken over $\varphi \in \mathrm{Aut}(\mathbf{D})$, the group of Möbius maps of the unit disk $\mathbf{D}$. The Möbius invariance of the Q_p norm is then a consequence of the well-known Möbius invariance of the Dirichlet integral. A good summary of recent research on Q_p spaces is Xiao's monograph [**10**].

Since every Möbius map φ can be written as $\varphi(z) = e^{i\theta}\varphi_a(z)$, where θ is real and

$$\varphi_a(z) = \frac{a - z}{1 - \overline{a}z}$$

2000 AMS *Mathematics subject classification.* Primary 30H05 and 46E15.
The first author was supported by NNSF-China Grant No. 10371069. The second author was partially supported by the U.S. National Science Foundation.
Received by the editors on October 13, 2005, and in revised form on November 21, 2005.

DOI:10.1216/RMJ-2008-38-1-329