

SECOND-ORDER STURM-LIOUVILLE BOUNDARY VALUE PROBLEM INVOLVING THE ONE-DIMENSIONAL p -LAPLACIAN

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ABSTRACT. In this paper, we prove the existence of at least three solutions for the Sturm-Liouville boundary value problem depending upon the parameter λ . Our main tool is a three critical points theorem given by Averna and Bonanno [3].

1. Introduction. In recent years, a great deal of work has been done in the study of the existence of multiple solutions of two-point boundary value problems, by which a number of physical and biological phenomena are described. For the background and results, we refer the reader to the monograph by Agarwal et al. and some recent contributions such as [2, 6–9].

Various fixed point theorems are applied to get interesting results, see for example, [6–9] and the references therein. Among them, Krasnosel'skii fixed point theorem, Leggett-Williams fixed point theorem, a five functionals fixed point theorem and fixed point theorems in cones are very frequently used.

In recent years, a three critical point theorem given by Ricceri [10] is also widely used and has been generalized by Averna and Bonanno [3]. Using the variational principle and the mountain pass theorem, Averna and Bonanno gave a definite interval, say $]1/\varphi_2(r), 1/\varphi_1(r)[$, in which λ lies, then $\Phi + \lambda\Psi$ has at least three critical points. Their result is as follows.

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