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FLAT EPIMORPHISMS AND A GENERALIZED KAPLANSKY IDEAL TRANSFORM

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ABSTRACT. We generalize the notion of the Kaplansky ideal transform $\Omega(I)$ to an ideal I in an arbitrary commutative ring R by defining $\Omega(I)$ as the localization of R with respect to a certain filter of ideals. It is shown that if the total ring of quotients of R is von Neumann regular, then $\Omega(I)$ is the ring of global sections over the open set $D(I)$. Additionally for such rings, we characterize when the open set $D(I)$ is an affine scheme in terms of the flatness of $\Omega(I)$.

1. Introduction. Let R be an integral domain with quotient field K. For an ideal I, the Nagata ideal transform $N_R(I) = \cup_{n\geq 0}(R :_K I^n) = \{q \in K : I^n \subseteq (R :_K q) \text{ for some } n \geq 0\}$, where $(R :_X q) = \{r \in X : rq \in R)$, has proven to be a very useful tool in various areas of commutative ring theory. Not only in its original application by Nagata in solving Hilbert's fourteenth problem, but also in the general study of overrings, see [**1, 2, 9, 10**].

However, once one leaves the realm of Noetherian rings, the Nagata transform apparently is not as useful a tool. For nonfinitely generated ideals I of an integral domain R, a variant of this transform has been studied and proven to be of significant value. The Kaplansky (ideal) transform of R with respect to an ideal I of R is the overring

$$\Omega_R(I) := \{q \in K : I \subseteq \mathrm{Rad}\,(R :_R q)\}.$$

Observe that $\Omega_R(I)$ is an overring of the Nagata ideal transform of I, with the two transforms equal if I is finitely generated.

As noted by Fontana in [**4**], localizing (or Gabriel) filters of ideals and generalized rings of quotients is a natural approach to the study of ideal transforms (see Section 2 for the definitions and basic results). Carrying this notion a step further, we define a generalized Kaplansky

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