ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 38, Number 1, 2008

ON MULTI-DIMENSIONAL SDES WITH LOCALLY INTEGRABLE COEFFICIENTS

V.P. KURENOK AND A.N. LEPEYEV

ABSTRACT. We consider the multi-dimensional stochastic equation

$$X_t = x_0 + \int_0^t B(s, X_s)\, dW_s + \int_0^t A(s, X_s)\, ds$$

where x_0 is an arbitrary initial value, W is a d-dimensional Wiener process and $B : [0, +\infty) \times \mathbf{R}^d \to \mathbf{R}^{d^2}$, $A : [0, +\infty) \times \mathbf{R}^d \to \mathbf{R}^d$ are measurable diffusion and drift coefficients, respectively. Our main result states sufficient conditions for the existence of (possibly, exploding) weak solutions. These conditions are some local integrability conditions of coefficients B and A. From one side, they extend the conditions from [**3**] where the corresponding SDEs without drift were considered. On the other hand, our results generalize the existence theorems for one-dimensional SDEs with drift studied in [**4**]. We also discuss the time-independent case.

1. Introduction. In this note we consider a stochastic equation of the form

$$X_t = x_0 + \int_0^t B(s, X_s)\, dW_s + \int_0^t A(s, X_s)\, ds, \quad t \geq 0, \tag{1.1}$$

where the coefficients $B : [0, +\infty) \times \mathbf{R}^d \to \mathbf{R}^{d^2}$, $A : [0, +\infty) \times \mathbf{R}^d \to \mathbf{R}^d$ are Borel measurable matrix- and vector-valued functions with $d \geq 1$, respectively, W is a d-dimensional Wiener process and $x_0 \in \mathbf{R}^d$ is an arbitrary initial vector.

2000 AMS *Mathematics subject classification.* Primary 60H10, 60J60, 60J65, 60G44.
Keywords and phrases. Multi-dimensional stochastic differential equations, locally integrable coefficients, Krylov's estimates, Wiener process, weak convergence.
Received by the editors on April 27, 2004, and in revised form on August 30, 2005.

DOI:10.1216/RMJ-2008-38-1-139