ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 4, Fall 1989

MIXED MODULES IN L*

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ABSTRACT. By assuming the set-theoretic hypothesis $V = L$ we show that, for a large class of rings R, there exist, for any regular not weakly compact cardinals κ, strongly κ-cyclic mixed R-modules having endomorphism algebra isomorphic to the split extension of the R-algebra A by the ideal of bounded endomorphisms provided A is free quâ R-module and $\kappa > |A|$.

1. Introduction. In this paper we deal with endomorphism algebras $E_R(G)$ of certain mixed R-modules G in the universe $V = L$. We shall always assume that R is a non-zero commutative ring with 1, with a given countable multiplicatively closed subset S of non-zero divisors. Let A be any fixed R-algebra which is S-reduced and S-torsion-free. (These and related concepts are defined in §2.) It has been established, working only in ZFC, that inter alia the following realization theorem holds; see [**1**] and [**2**].

THEOREM. *If A is an S-reduced, S-torsion-free R-module then there exists a mixed R-module G with $E_R(G) = A \oplus \mathrm{Bd}(G)$. (Here and throughout the paper $\mathrm{Bd}(G)$ will denote the ideal of bounded endomorphisms of G; $\phi \in \mathrm{Bd}(G)$ if and only if there is an $s \in S$ with $(G\phi)s = 0$.)*

Indeed the results can be extended to derive arbitrarily large rigid systems and semi-rigid proper classes (i.e., classes which are not sets.) Assuming $V = L$ we can sharpen these results considerably by imposing only slightly stronger conditions on the algebra A. In this context cyclic A-modules will either be copies of A or torsion A-modules A/sA for some $s \in S$. Recall that, in general, a module is said to be Σ-cyclic if it is a direct sum of cyclic modules. Observe that Σ-cyclic modules are reduced. A module is κ-cyclic, for some cardinal κ, if any

This paper was written under contract SC115/84 from the National Board for Science and Technology (Ireland).
Received by the editors on May 28, 1986, and in revised form on February 9, 1987.
AMS *Subject classification*: 20K30, 20K21.