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ON THE MÖBIUS FUNCTION OF A FINITE GROUP

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0. Our purpose is to present an elementary, uniform, and self-contained treatment of a growing body of knowledge about the Möbius function μ associated with the subgroup lattice of a finite group G. Our principal result (Theorem 5.1) has as a special case the following recent theorem of Thévenaz (whose preprint [**25**] came to our attention only after we had completed this work in the early months of 1986): *If an integer n divides $|G|$, then it also divides*

$$(*) \qquad \sum_{\substack{X \leq G \\ |X| \text{ divides } n}} \mu(X).$$

This generalizes a result of Brown's [**4**], which covers the case where $n = |G|_p$, the order of a Sylow p-subgroup of G. Another easy consequence of our Theorem 5.1 is and elementary proof of the theorem of Frobenius about the number of solutions of the equation $x^n = 1$ in a group. We have also included a proof of a conjecture of Thévenaz (see 4.2 of [**26**]).

To emphasize the unity of our approach the main exposition is couched in elementary algebraic language and avoids reference to the Burnside ring, to the theory of projective modules, and to topological methods, all of which have been used by Thévenaz and other authors in this context. For the sake of completeness and directness we have included easier proofs of some known results, and to this extent our presentation is partly expository. In a final section we describe some topological connections and interpretations of our results.

1. Historical Introduction. In his 1936 paper on Eulerian functions Philip Hall [**13**] defined the Möbius function on a subset of a

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