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ON THE ANALYTICITY OF THE HOMOLOGY

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Introduction . Let W be a real analytic manifold and $\{\alpha\} \in H_P(W, Z_2)$. We shall say that $\{\alpha\}$ is analytic if there exists a compact analytic subset S of W such that: $\{\alpha\} =$ (fundamental class of S). The purpose of this short paper is to prove

THEOREM 1. *Let W be a paracompact real analytic manifold. Then any homology class $\{\alpha\} \in H_P(W, Z_2)$ is analytic.*

We remark that a similar result fails to hold in the real algebraic case (see [**2**]).

1. Definitions and well known facts. Let V, W be two differentiable (i.e., $\mathbf{C}^{\infty}$) manifolds. Then, on the set $M(V, W)$ of differentiable maps $f : V \to W$, we shall consider the Whitney topology (see [**5**, p. 42]).

In the following we shall use the known result: if $f \in M(V, W)$, then there exists a neighborhood U, in the $\mathbf{C}^{O}$ topology, of f such that any $g \in U$ is homotopic to f (see [**7**]).

By a real algebraic variety we shall mean an affine real algebraic variety. A regular variety will be called an algebraic manifold. An algebraic map is the restriction of a rational map.

In the following we shall need

LEMMA 1. *Let $V \subset \mathbf{R}^n, W \subset \mathbf{R}^q$ be two real algebraic manifolds and $V \xrightarrow{\varphi} W$ be a differentiable map. If V is compact and bordant to ϕ, then, for any $\varepsilon > 0$, there exists an algebraic submanifold $V' \subset \mathbf{R}^{n+q}$, an analytic isomorphism $V' \xrightarrow{\pi} V$ and an algebraic map $\varphi' : V' \to W$ such that:*

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