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OPEN MORPHISMS OF REAL CLOSED SPACES

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Introduction. In [**8**] and [**9**] it is shown that the theory of semi-algebraic spaces developed by H. Delfs and M. Knebusch (see [**3**, **4**]) can be extended by the theory of real closed spaces much as Grothendieck's theory of schemes extends the classical theory of varieties. In this paper the systematic study of real closed spaces is continued by looking at open and generalizing morphisms. The reason for the interest in these morphisms is that in the theory of schemes openness implies that there is some regularity in the behavior of the fibres of a morphism (see [**6**]).

In §1, some basic properties of open, generalizing, universally open, universally generalizing morphisms are collected. In the theory of schemes, there are connections between algebraic properties of a morphism and openness. For example, a locally finitely presented flat morphism of schemes is universally open [**5**, I.7.3.10]. Inspired by this result, algebraic properties of generalizing morphisms are investigated in §2. A valuative characterization of universally generalizing morphisms is in §3. Finally, in §4, the fibres of affine finitely presented morphisms of real closed spaces are studied. The fibres of these morphisms are affine semi-algebraic spaces. So, from semi-algebraic geometry there are numerous numerical invariants of these spaces (for example: dimension, number of connected components, Betti numbers). It is easy to construct examples of morphisms of semi-algebraic spaces in which these invariants of the fibres change very drastically. Openness (in connection with other hypotheses) brings some measure of regularity into the behavior of the fibres as far as dimension and number of connected components are concerned. For example, if $f : X \to Y$ is affine, finitely presented and universally open, then $\dim f^{-1}(y') \geq \dim f^{-1}(y)$ if y is a specialization of y'.

1. Open morphisms. The basic notions of open and generalizing morphisms of real closed spaces are adapted from the theory of schemes:

DEFINITION 1. Let $f : X \to Y$ be a morphism of real closed spaces.