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FINITE SPACES OF SIGNATURES: RESEARCH ANNOUNCEMENT

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Mulcahy's Spaces of Signatures [**3**] provide an abstract setting for Becker and Rosenberg's reduced Witt rings of higher level [**1**], along the same lines as Marshall's Spaces of Orderings setting for the ordinary reduced Witt ring.

This note concerns a generalization of Marshall's result that a finite Space of Orderings arises in connection with a field [**2**].

DEFINITION. A Space of Signatures (SOS) is a pair (X, G), where G is an abelian group of even exponent, and $X \subseteq G^* = \mathrm{Hom}\,(G, \mu)$ (μ being the complex roots of 1), which satisfies certain axioms (see [**3**]).

(X, G) is said to be realizable when $(X, G) = (X_T, \dot{K}/\dot{T})$ for a preordered field $\{K, T\}$.

Let (X, G) be a SOS such that $|G| = 2^s$ for some $s \in \mathbf{N}$. Our main result is

THEOREM 1. (X, G) *is realizable.*

The general idea of the proof of Theorem 1 is Marshall's: We show that (X, G) can be built up from smaller SOS's, and since the building operations preserve realizability, we can use induction.

DEFINITION. (X, G) is said to be a *group extension* of an SOS (X_0, G_0) if $G_0 \subseteq G$ and $X = \{\sigma \in G^* : \sigma|_{G_0} \in X_0\}$. If we just say that (X, G) is a group extension, we mean that (X, G) is a group extension of an SOS (X_0, G_0) where $G_0 \neq G$.

An SOS (X, G) is said to be a *direct sum* of the SOS's (X_1, G_1) and

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