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THE REPRESENTATION THEOREM FOR SPACES OF SIGNATURES

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As shown in [**6**], there is a commutative diagram of form theories and generalization maps as follows:

$$\begin{array}{ccc} \text{HIGHER LEVEL FORMS} & \longrightarrow & \text{SPACES OF SIGNATURES} \\ \uparrow & & \uparrow \\ \text{QUADRATIC FORMS} & \longrightarrow & \text{SPACES OF ORDERINGS} \end{array}$$

On the left we have the reduced theory of quadratic forms over fields, along with Becker and Rosenberg's extension to a reduced higher level form theory [**2**]. The abstract theory of Spaces of Orderings [**5**] facilitates a unified treatment of reduced quadratic forms over fields, skew fields and semi-local rings. The higher level theory can also be carried out over skew fields [**8**], and Spaces of Signatures [**6, 7**] allow for a simultaneous generalization of all of these theories.

Associated to each Space of Signatures (X, G) is a reduced Witt ring $W(X)$, which embeds naturally in a ring of continuous functions $C(X, \mathbf{C})$. The problem of characterizing this subring of $C(X, \mathbf{C})$ was solved in [**7**], under a restrictive 2-power assumption, by extending the representation theorems of [**1, 2, 4**] and [**5**]. In this note we explain how this may be done for any Spaces of Signatures, without the 2-power assumption. Detailed proofs will appear in a forthcoming paper.

Let G be an abelian group of finite (even) exponent, and set $G^* = \mathrm{Hom}_{\mathbf{Z}}(G, \mu)$, with the usual compact-open topology (μ being the complex roots of unity, with the discrete topology). Fix a nonempty subset X of G^*. An m-dimensional form f is an m-tuple $\langle a_1, \ldots, a_m \rangle (a_i \in G)$. The notions of equivalence, isometry, isotropy, represented sets, and sums of forms are defined just as for Spaces of Orderings (see [**6**]).

DEFINITION. (X, G) is a *Space of Signatures* when these axioms hold:

S_0: If $\sigma \in X$, then $\sigma^k \in X$ for all odd k.