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EVEN POSITIVE DEFINITE UNIMODULAR QUADRATIC FORMS OVER REAL QUADRATIC FIELDS

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In spite of the numberous connections between even positive definite unimodular quadratic forms (henceforth referred to as even unimodular lattices) over $\mathbf{Q}$ with other subjects (e.g., finite group theory, geometry of numbers, combinatorial coding and design theories, automorphic functions, the explicit classification of these lattices has only been fully determined for a few cases, the most celebrated of them being undoubtedly the Leech-Niemeier-Witt [**4**] solution for the 24-dimensional $\mathbf{Z}$-lattices. We discuss here some results on the classification of even unimodular lattices over some real quadratic number fields.

Let $F = \mathbf{Q}(\sqrt{p}), p$ a prime, $R = \mathrm{int}(F), d = d_F$ the field discriminant, and $e = e_F$ the fundamental unit. Let the rank of such an R-lattice be m. Then, m is even. If $p \equiv 3(\bmod 4)$, write $p = -1+4t, t > 0$. Clearly,

$$\begin{bmatrix} 2 & \sqrt{p} \\ \sqrt{p} & 2t \end{bmatrix}$$

defines a binary even unimodular lattice over F. On the other hand, if $p \equiv 1(\bmod 4)$ then it is not difficult to see from the local dyadic structures of the lattices that m must satisfy $m \equiv 0(\bmod 4)$. The same holds for $p = 2$.

I. Analytic mass formula. One way to get a crude estimate for the class number of such lattices is via Siegel's anlytic mass formula. Let $M_m(F)$ be the Minkowski-Siegel mass of the genus of an R-lattice over F of rank $m \equiv 0(\bmod 2)$ and determinant $+1$. One then has the following formula whose proof is analogous to that given in [**1**] for $\mathbf{Q}(\sqrt{5})$.

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