

A DECISION METHOD FOR CERTAIN ALGEBRAIC GEOMETRY PROBLEMS

HAI-PING KO AND SHANG-CHING CHOU

ABSTRACT. We present a mathematical theorem in algebraic geometry. The theorem implies a decision method for the membership of the set of all the polynomials which fix a certain type of algebraic variety denoted by V^* by Wu Wen-tsün. The theorem is a generalized form of similar theorems developed by Ritt, Wu, and the above authors. And the decision procedure is a natural extension of similar decision procedures introduced by Ritt, Wu, and the second author.

Wu Wen-tsün's method of mechanical theorem proving in geometry is complete for certain elementary geometry problems involving equality only. For the corresponding algebraic geometry problem, the method is complete for problems with an algebraically closed field as the associated field, but not complete for the above type of problems with the field of rational numbers (\mathbf{Q}) or the field of real numbers (\mathbf{R}) as the associated field. As suggested by Wu in 1982, the second author shows a condition for Wu's method to be complete for the above problems with R as associated field. We now show a more general condition for Wu's method to be complete for the above algebraic geometry problems with any field as the associated field.

Background. The algebraic problem to be presented here is extracted from a study of algebraic methods of automated geometry theorem proving.

Research in automated geometry theorem proving has been motivated by such visions as: (1) providing a mathematical tool for education in geometry, (2) studying the basic needs of an intelligent system, and (3) advancing the technology of robotics and computer vision. Proposed methods of automated geometry theorem proving can be classified as either logical or algebraic. For instance, the methods introduced by Tarski [10] and Wu Wen-tsün [12, 13] are considered as algebraic. The method introduced by Wu Wen-tsün has been considered as a breakthrough success since the time of Tarski. It

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