ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 3, Summer 1989

IDEAL CLASS GROUPS OF WITT RINGS

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Let F be a formally real field with only finitely many orderings. Let R denote the Witt Ring of F. In [**1**] we gave necessary and sufficient conditions for every ideal of R containing an odd dimensional form to be principal. The wish to place this result in the more natural context of multiplicative ideal theory led to the problem of computing the ideal class group $C(R)$ of R. Details will appear elsewhere.

DEFINITION. An element $a \in R$ is *regular* if it is not a zero-divisor and *strongly regular* if it is odd dimensional. An ideal $I \subset R$ is (*strongly*) *regular* if it contains an element which is (strongly) regular.

PROPOSITION 1. *Let $I \subset R$ be a strongly regular ideal. Then:*

(1) *I is a unique (finite) product of prime ideals;*

(2) *I has a unique primary decomposition.*

SKETCH OF PROOF. If R is not reduced, then R is a Prüfer Ring [**4**], and so (1) follows. (2) follows from (1) by the identity $(I+J)(I\cap J) = IJ$ (for ideals I, J one of which is regular) which holds in Prüfer Rings [**3**].

If R is reduced, then (2) follows from previous work on primary decomposition [**1**]. And (1) is deduced from (2) by a standard primary decomposition argument. □

PROPOSITION 2. *Let $[I]$ denote the class of a regular ideal I in the ideal class group $C(R)$. Then there exists a strongly regular ideal J such that $[I] = [J]$.*

PROOF. We may assume $I \subset IF$. Then I_{IF} is principal, generated by a regular element $a \in I$. Now $I = (a, b)$ for some $b \in R$ by [**2**]. Since