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NASH FUNCTIONS AND THE STRUCTURE SHEAF

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Let $\mathbf{R}$ be a real closed field and $U \subset \mathbf{R}^n$ an open semialgebraic (s.a.) set. A Nash function over U is a function of class C^∞ and s.a. If $\mathbf{R} = \mathbb{R}$, this definition agrees with the usual one [**1**, Chapter 8].

If $A = \mathbf{R}[X_1, \ldots, X_n]$ and $\tilde{U}$ is the constructible set in $\operatorname{Spec}_r A$ associated to U, the ring of Nash functions over U is canonically isomorphic to the ring $\mathcal{N}_A(\tilde{U})$ of global sections over $\tilde{U}$ of the structure sheaf of $\operatorname{Spec}_r A$. This is a consequence of the Artin-Mazur description of Nash functions and the behavior of the operator $\sim$. For this result and other basic properties of the structure sheaf see M.-F. Roy's article [**8**].

Now, let $V \subset \mathbf{R}^n$ be an algebraic variety (not necessarily smooth) and let A be its coordinate ring. In the above quoted article, we observe that if N_V is the sheaf obtained by restriction and identification of elements of $\mathcal{N}_{R[X_1,\ldots,X_n]}$ over $\operatorname{Spec}_r A$, this sheaf does not necessarily coincide with $\mathcal{N}_A$. Moreover, an example of a variety for which these sheaves differ is given, the study of the relationship between them is proposed and it is conjectured (for $\mathbf{R} = \mathbb{R}$) that the set of points of V, for which the stalks of both sheaves are isomorphic, is the set of quasi-regular points of V in Tognoli's sense.

To answer these questions, our first results are the following theorems.

THEOREM 1. ([**2**, II. 1.5]. or [**3**, 1.7]) *For every $\alpha \in \operatorname{Spec}_r A$ the stalk $N_{V,\alpha}$ is naturally isomorphic to $\mathcal{N}_{A,\alpha}/\operatorname{rad}_r(0)$.*

THEOREM 2. ([**2**, II. 2.1.], or [**3**, 2.1]) *Let $x \in V$. The following statements are equivalent:*

(i) *x is quasi-regular (i.e., the complexification of the Nash germ V_x coincides with the complex Nash germ at x of the algebraic complexification V_c of V);*

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