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ORDERINGS AND VALUATIONS ON *-FIELDS

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1. Introduction to orderings. Let $(D, *)$ be a $*$-field; that is, a skew field D with an involution $*$ (an anti-automorphism of order 2). Beginning with a definition of Baer, at least four different notions of ordering have been proposed for D [**1**, **3**, **5**, **6**] with various relationships among them. In this paper we shall work with three of these notions, giving a description of the liftings to D of orderings of the residue class field of a valuation on D. The theorems proved below generalize the commutative theory for orderings and semiorderings (D commutative, $*$ equal to the identity) found in [**7**, Chapter 7]. The pioneering work with valuations on $*$-fields was done by Holland [**4**, **5**]. We shall find it convenient to slightly modify some of his definitions in order to arrive at a complete theory of how orderings lift.

Some notation that we shall use throughout this paper includes writing $\mathrm{S}(D)$ for the set of symmetric elements in D, namely $\{d \in D \mid d = d^*\}$, $\Pi S(D)$ for the set of all nonzero products of elements from $\mathrm{S}(D)$ and, for any subset A of D, writing $A^\times$ for the collection of nonzero elements of A.

DEFINITION 1.1. A *Baer ordering* on $(D, *)$ is a subset P of $\mathrm{S}(D)$ satisfying

(a) $P + P \subseteq P$;

(b) $1 \in P$ and, for any nonzero element $d \in D$, $dPd^* \subseteq P$;

(c) $P \cup -P = S(D)^\times$; and

(d) $P \cap -P = \emptyset$

For D commutative with $* =$ identity, a Baer ordering is a semiordering as defined in [**7**]. The next two definitions give different ways of extending the standard notion of ordering.

DEFINITION 1.2. A *Jordan ordering* of $(D, *)$ is a Baer ordering P