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DISCRETELY VALUED FIELDS WITH INFINITE u-INVARIANT: RESEARCH ANNOUNCEMENT

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Herbert Gross (see [**1**, p. 3]) has raised the following problem:

PROBLEM (G). *Is there any commutative field which admits no anisotropic* $\aleph_0$*-quadratic form but which admits, for each* $n \in N$, *some anisotropic form in* n *variables?*

There has been some related work in infinite dimensional anisotropic quadratic forms (see, e.g., Meissner [**2**] and [**3**]), but Gross's problem is still open. In this note we contribute a partial solution to this problem, we prove that if the answer to it is positive then there has to be a field of a very specific kind that fulfills those conditions. Also, we introduce some nonarchimedean analysis techniques to study the discretely real-valued commutative fields for which there is some anisotropic $\aleph_0$-quadratic form. Proofs will be published elsewhere.

Let us first introduce some notations and terminology. The u-invariant of a field F, $u(F)$, as defined by Kaplansky, is the maximum (when it exists) of the set of natural numbers n such that there is an n-dimensional linear space E over F and an anisotropic quadratic form on E. If that maximum does not exist, we say that $u(F)$ is (*weakly*) *infinite*, and in case there is an infinite-dimensional vector space E over F with an anisotropic quadratic form, then we say that $u(F)$ is *strongly infinite*.

We will denote by K any commutative field which is endowed with a nontrivial discrete real valuation (that is, that has as its value group a discrete subgroup of $\mathbf{R}^+$) and is complete for the associated distance. We will call k the residue class field of K, and shall always assume that $\operatorname{char} k \neq 2$, that is to say, that K is nondyadic.

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