ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 3, Summer 1989

THE BEHAVIOR OF THE ν-INVARIANT OF A FIELD OF CHARACTERISTIC 2 UNDER FINITE EXTENSIONS

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ABSTRACT. Let F be a field of characteristic 2. We define $\nu(F)$ as the smallest integer n such that any n-fold quadratic Pfister form over F is isotropic. If L/F is any finite extension, we prove $\nu(F) \leq \nu(L) \leq \nu(F)+1$. The corresponding question for fields of characteristic $\neq 2$ is still open.

1. Introduction. The ν-invariant of a field F of characteristic $\neq 2$ was introduced in [**2**] as the number $\nu(F) = \text{Min}\{n \mid I^n(F)$ is torsion free$\}$, where $I(F)$ denotes the maximal ideal of even dimensional quadratic forms over F in the Witt ring $W(F)$. If F is non real, then $\nu(F)$ is the smallest integer n such that any n-fold Pfister form over F is isotropic. Similarly, if F is a field of characteristic 2, let $W_q(F)$ be the Witt group of non singular quadratic forms over F and $W(F)$ the Witt ring of non singular symmetric bilinear forms over F. It is well known that $W_q(F)$ is a $W(F)$-module under the operation $b \cdot q(x \otimes y) = b(x,x)q(y)$ for any $x \in V =$ space of the bilinear form b, $y \in W =$ space of the quadratic form q. If $I(F) \subset W(F)$ is the maximal ideal of even-dimensional bilinear forms, then the chain of submodules $W_q(F) \supset IW_q(F) \supset I^2W_q(F) \supset \cdots$ plays an important role in the knowledge of the module structure of $W_q(F)$. If $a_1,\ldots,a_n \in F^*$, $b \in F$, then the quadratic n-fold Pfister form $\langle 1,a_1\rangle \ldots \langle 1,a_n\rangle[1,b]$ is a typical generator of $I^nW_q(F)$, where $\langle 1,a\rangle$ denotes the symmetric bilinear form $U^2 + aV^2$ and $[1,b]$ denotes the quadratic form $X^2+XY+bY^2$. We shall usually write $\langle\langle a_1,\cdots,a_n,b]]$ instead of $\langle 1,a_1\rangle \cdots \langle 1,a_n\rangle[1,b]$. (We refer to [**1, 3**] for general facts on quadratic forms in characteristic 2). We define now, as in [**2**], the ν-invariant of a field F of characteristic 2 as

$$\nu(F) = \text{Min}\{n \mid I^nW_q(F) = 0\}, \tag{1.1}$$

i.e., $\nu(F)$ is the smallest integer n such that any n-fold Pfister form over F is isotropic.

Supported partially by CHI-84-004, PNUD and FONDECYT.
Received by the editors on October 29, 1986 and in revised form on April 15, 1987.