ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 3, Summer 1989

ON QUADRATIC FORMS AND GALOIS COHOMOLOGY

JÓN KR. ARASON, RICHARD ELMAN
AND BILL JACOB

1. Introduction. In the articles [**4-8**], we investigate certain problems that arise naturally in the algebraic theory of quadratic forms and make progress toward their solution. The basic motivation for studying these problems is the interaction between the algebraic theory of quadratic forms over fields and Galois cohomology. In this survey, we discuss some of these problems and results obtained so far.

Throughout, F will denote an arbitrary field of characteristic different from two. We denote by WF the *Witt ring* of equivalence classes of quadratic forms over F and by IF the ideal of even dimensional forms in $W\,F$. (See [**23**] or [**29**] for terminology from quadratic form theory.) We let $I^nF = (IF)^n$ and denote by $GW\,F$ the associated graded ring, i.e., $GW\,F := \oplus_{i=0}^{\infty}\overline{I}^n F = \oplus_{i=0}^{\infty} I^nF/I^{n+1}F$. We call $GW\,F$ the *graded Witt ring of* F. We let $H^nF := H^n(G_F, \mathbf{Z}/2\mathbf{Z})$, where G_F denotes the absolute Galois group of F and $H^*F := \oplus_{i=0}^{\infty} H^nF$. With the cup product, H^*F is a graded ring, the '*full*' mod 2 *cohomology ring* of F. (See [**28**] for terminology from Galois cohomology theory.) One would like to determine the relationship between the $GW\,F$ and H^*F.

The classical invariants give rise to homomorphisms:

$$\begin{array}{ll} e_F^0 : W\,F \to H^0F & \text{dimension mod } 2 \\ e_F^1 : IF \to H^1F & \text{(signed) discriminant} \\ e_F^2 : I^2F \to H^2F & \text{Clifford invariant.} \end{array}$$

It is natural to ask if this sequence of invariants continues. By analogy with e_F^0, e_F^1, and e_F^2, the general invariant should be a homomorphism $e_F^n : I^nF \to H^nF$ that maps (the class of) the *n-fold Pfister form* $\langle 1, -a_1\rangle \cdots \langle 1, -a_n\rangle$ to $(a_1) \cup \cdots \cup (a_n)$. Here $(a) \in H^1F \simeq \dot{F}/\dot{F}^2$ corresponds to the square class of a. By [**1**] or [**12**], this assignment is indeed a well-defined map on the set of n-fold Pfister forms in $W\,F$.

Supported in part by the National Science Foundation.
Received by the editors on October 1, 1986 and in revised form on May 2, 1987.