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A NEW APPROACH TO THE STUDY OF HARRIS TYPE MARKOV OPERATORS

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Harris operators are generalizations of Markov matrices. It is our purpose to present an elementary discussion of the theory of Harris operators. In Chapter 1 we introduce most of the results about Markov operators to be used later. In Chapter 2 we study Orey's Lemma. And in the rest of the paper we use Orey's Lemma to give elementary proofs of Harris' Theorem, Ornstein-Metivier-Brunel Theorem, Doeblin's theorem, and Pointwise Convergence of uP^n.

1. Introduction. We shall use the definitions and notation of [**3**] and [**4**].

Recall that *if* λ *is* σ *finite measure on* (X, Σ), *then a Markov operator*, P, *is a linear operator on* $L_\infty(X, \Sigma, \lambda)$ *satisfying*

$$P1 \leq 1; \quad f \geq 0 \Rightarrow Pf \geq 0; \quad f_n \downarrow 0 \Rightarrow Pf_n \to 0$$

All inequalities, here and elsewhere are in the a.e. sense. Denote $\langle u, f\rangle = \int uf d\lambda; u \in L_1$ and $f \in L_\infty$. The dual operator acts on L_1 by $\langle uP, f\rangle = \langle u, Pf\rangle; u \in L_1$ and $f \in L_\infty$.

We may extend P, by monotone continuity, so that Pf and uP are defined for *all* non-negative measurable functions [**3**, Chapter I].

THEOREM 1.1. *Let* P *be conservative and ergodic. Then:*

(1) $P1 = 1$.

(2) $f \geq 0, Pf \leq f \Rightarrow f = \text{Const}$.

(3) $f \geq 0, \ f \neq 0 \Rightarrow \Sigma P^n f \equiv \infty$.

(4) $u \geq 0, \ u \neq 0 \Rightarrow \Sigma uP^n \equiv \infty$.

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