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(0.3) INTERPOLATION ON THE ZEROS OF $\pi_n(x)$

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1. Introduction. Balázs' and Turán's work [**1**] on (0,2) interpolation in 1957 led to considerable interest in the general problem of Birkhoff interpolation. However, in spite of the recent classic book on this subject by G.G. Lorentz et al. [**2**], the problem of (0,3) interpolation on the zeros of $\pi_n(x)$ seems to have been ignored. Similarly, although we know [**2**, p. 10] that (0,2,3) interpolation is regular on any real distinct nodes, i.e., is always uniquely solvable, there is no known formula for the explicit expression for the interpolant, except in the trigonometric case on equidistant nodes.

Recently Varma has found some quadrature formulae using values and third derivatives of $\pi_n(x)$ together with values of the first derivatives at ± 1 on using his method in [**3**]. However, his approach is not via interpolatory formulae. In view of this, we propose to show that (0,3) interpolation is regular for $n \geq 4$ on the zeros of $\pi_n(x)$ and to give the explicit formulae for the fundamental polynomials. (For $n < 3$, the problem is not regular because Polya conditions are not satisfied and for $n = 3$, the problem is trivial.) It turns out that the quadrature formula of Varma can be obtained by integrating the polynomial of (0,3) interpolation. The methods used here show that the problem of $(0,1,\ldots,r-3,r)$ on zeros of $\pi_n(x)$ is regular for any positive integral $r \geq 3$.

In §2, we give the preliminaries and state the main results. The proof of Theorem 1 is given in §3 and the fundamental polynomials are derived in §4. §5 comprises the proof of Theorem 2 and the fundamental polynomials for the (0,3) case are given in §6. In §7, we apply the results to derive a quadrature formula.

2. Preliminaries and main results. It is known that the polynomials $\pi_n(x)$ satisfy the differential equation

$$(2.1) \qquad (1-x^2)y'' = -n(n-1)y, \quad n \geq 2.$$

For $n = 0$ and 1, $\pi_0(x) = 1$, $\pi_1(x) = x$ and $\pi_n(x) = (1-x^2)P'_{n-1}(x)$ where $P_n(x)$ denotes the Legendre polynomial of degree n with $P_n(1) = 1$.