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CONVERSE RESULTS IN THE THEORY OF EQUICONVERGENCE OF INTERPOLATING RATIONAL FUNCTIONS

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1. Introduction. Since the first extension of Walsh's theorem in 1981 [**1**], there have been in the last few years a number of direct theorems on the theory of equiconvergence of certain schemes of interpolatory polynomial sequences. A recent paper of Saff and Sharma [**3**] also gives some direct theorems, but it deals with the equiconvergence of two schemes of rational interpolants. Our object in this paper is to obtain a sort of converse of this theorem on the lines of a corresponding theorem due to Szabados [**4**] which is related to the Lagrange interpolant and the Taylor sections of an analytic function.

Let $f \in A_\rho$ (the class of functions analytic in $|z| < \rho$ but not in $|z| \leq \rho$, $\rho > 1$). As usual π_s will denote the class of polynomials of degree $\leq s$. For a given $\sigma > 1$ and for a fixed integer $m \geq -1$, let

$$(1.1)\quad r_{n+m,n}(z,f) := B_{n+m,n}(z,f)/(z^n - \sigma^n),\ B_{n+m,n}(z,f) \in \pi_{n+m},$$

interpolate $f \in A_\rho$ in the $n+m+1$ roots of unity. If, for a positive integer l, we set

$$(1.2)\qquad \Delta^{\sigma}_{l,n,m}(z;f) = R_{n+m,n}(z,f) - \sum_{\nu=0}^{l-1} r_{n+m,n}(z,f,\nu),$$

where $r_{n+m,n}(z,f,\nu)$ are certain rational functions given by (2.1) and (2.3), then Saff and Sharma showed that if $\sigma \geq \rho^{l+1}$, then

$$(1.3)\qquad \lim_{n\to\infty} \Delta^{\sigma}_{l,n,m}(z,f) = 0$$

for $|z| < \rho^{l+1}$. And if $\sigma < \rho^{l+1}$, then (1.3) holds for all $z \in \mathbf{C}$ with $|z| \neq \sigma$. Moreover, this result is sharp in the sense that the region of convergence cannot be improved.

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