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EXTENSIONS OF THE HEISENBERG GROUP AND COAXIAL COUPLING OF TRANSVERSE EIGENMODES

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1. Optical fiber communication. Lightwave electronics, including optical communication via silica fibers and optoelectronic devices, has become one of the most promising fields of applied physics and electrical engineering since the laser first appeared.

The advantages of optical fiber communication are among others:

- extremely low loss of the optical signals over a wide range of wavelengths (less than 1dB/km, corresponding to a 25% loss per km)

- immense bandwith (1 and 100 GHz, respectively, for multimode and single-mode fibers over 1 km) that makes it possible to use extremely short pulses.

Characteristic of the progress in lightwave communication technology is the enormous reduction of the transmission loss of optical fibers accomplished in the last decade as illustrated by the diagram below.

One of the most important factors that helped make optical fiber communication a reality is the invention of the light-emitting diode (LED) and the semiconductor injection laser. The coupling between lasers and optical fibers causes some power loss which is described by the coupling coefficients of the various modes. The main purpose of the present paper is to calculate the coupling coefficients of the quantized transverse eigenmodes excited in coaxial circular and rectangular laser resonators and optical waveguides in terms of Krawtchouk polynomials evaluated at Gaussian beam parameters. The method we will present is based on non-commutative harmonic analysis, specifically, on the representation theory of various (three-step nilpotent and solvable) group extensions of the real Heisenberg two-step nilpotent Lie group $\tilde{A}(\mathbf{R})$.

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