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SHARP LOWER BOUNDS FOR A GENERALIZED JENSEN INEQUALITY

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Dedicated to Professor A. Sharma,
on his retirement from the University of Alberta.

1. Introduction. Our motivation for the research in this paper arose from two recent papers by Beauzamy and Enflo [**2**] and Beauzamy [**3**], which are connected with polynomials and the classical Jensen inequality. To describe their results, let $P(z) = \sum_{j=0}^{m} a_j z^j (= \sum_{j=0}^{\infty} a_j z^j$ where $a_j := 0$ for $j = m+1, m+2, \dots)$ be a complex polynomial $(\not\equiv 0)$, let d be a number in (0,1), and let k be a nonnegative integer. Then (cf. [**2, 3**]), $P(z)$ is said to have *concentration d at degrees at most k* if

$$\sum_{j=0}^{k} |a_j| \geq d \sum_{j=0}^{\infty} |a_j|. \tag{1.1}$$

(Later, we shall discuss functions which are *not* polynomials, yet for which (1.1) holds. This accounts for our use of the symbol, ∞, in (1.1).)

Beauzamy and Enflo showed (cf. [**3**, Theorem 1]) that there exists a constant $C_{d,k}$, depending only on d and k, such that, for any polynomial $P(z)$ satisfying (1.1), it is true that

$$\frac{1}{2\pi}\int_0^{2\pi} \log|P(e^{i\theta})|d\theta - \log\Big(\sum_{j=0}^{\infty} |a_j|\Big) \geq C_{d,k}. \tag{1.2}$$

For our purposes here, $C_{d,k}$ will denote the *largest* such constant possible in (1.2), i.e.,

$$C_{d,k} := \inf\Big\{\frac{1}{2\pi}\int_0^{2\pi} \log|P(e^{i\theta})|d\theta - \log\Big(\sum_{j=0}^{\infty}|a_j|\Big) : P(z) \text{ is a polynomial satisfying (1.1)}\Big\}. \tag{1.3}$$

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