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DIVIDED DIFFERENCE OPERATORS AND CLASSICAL ORTHOGONAL POLYNOMIALS

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ABSTRACT. In an earlier paper J. Wilson and I introduced a divided difference operator that plays the same role for the ${}_4\varphi_3$ orthogonal polynomials that the derivative does for Jacobi polynomials. Here this operator is used to give a new derivation of the connection coefficient result of L.J. Rogers.

1. Introduction. L.J. Rogers introduced a very attractive set of polynomials in [**10**]. To define them take q fixed with $0 < |q| < 1$. Set

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k) \tag{1.1}$$

and

$$(a;q)_n = (a;q)_\infty/(aq^n;q)_\infty. \tag{1.2}$$

Then, following Rogers (but using a slightly different notation), consider the generating function

$$\frac{(\beta re^{i\theta};q)_\infty(\beta re^{-i\theta};q)_\infty}{(re^{i\theta};q)_\infty(re^{-i\theta};q)_\infty} = \sum_{n=0}^{\infty} C_n(\cos\theta;\beta|q)r^n. \tag{1.3}$$

The q-binomial theorem is

$$\frac{(ar;q)_\infty}{(r;q)_\infty} = \sum_{n=0}^{\infty}\frac{(a;q)_n}{(q;q)_n}r^n. \tag{1.4}$$

Using this in (1.3) gives

$$\begin{aligned} C_n(\cos\theta;\beta|q) &= \sum_{k=0}^{n}\frac{(\beta;q)_{n-k}(\beta;q)_k}{(q;q)_{n-k}(q;q)_k}\, e^{i(n-2k)\theta} \\ &= \sum_{k=0}^{n}\frac{(\beta;q)_{n-k}(\beta;q)_k}{(q;q)_{n-k}(q;q)_k}\cos(n-2k)\theta. \end{aligned} \tag{1.5}$$

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