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PROXIMINALITY IN $L_p(S, Y)$

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Introduction. Throughout this work (S, Σ, μ) will be a finite measure space and Y a Banach space. For $1 \leq p < \infty, L_p(S, Y)$ is the Banach space consisting of strongly measurable functions $f : S \to Y$ such that $\int_S ||f(s)||^p ds$ is finite. In this case

$$||f||_p = \left\{ \int_S ||f(s)||^p \, ds \right\}^{1/p}.$$

Occasionally we shall consider the space $L_\infty(S, Y)$ which consists of all strongly measurable functions $f : S \to Y$ such that $\operatorname{ess\,sup} \{||f(s)|| : s \in S\}$ is finite. Then

$$||f||_\infty = \operatorname{ess\,sup}\{||f(s)|| : s \in S\}.$$

A typical example of the questions we investigate here is the following. Suppose H is a proximinal subspace of Y. Does it follow that $L_p(S, H)$ is a proximinal subspace in $L_p(S, Y)$? By way of introduction we indicate some results which are easy consequences of known theorems about the structure of $L_p(S, Y)$. The two key results are as follows:

THEOREM 1. *Let (S, Σ, μ) be a finite measure space, $p \in [1, \infty)$ and Y be a Banach space. Then $L_p(S, Y)^* = L_q(S, Y^*)$, where $p^{-1} + q^{-1} = 1$, if and only if Y^* has the Radon-Nikodym property with respect to μ.*

THEOREM 2. *Let (S, Σ, μ) be a finite measure space and Y a uniformly convex Banach space. Then, for $1 < p < \infty, L_p(S, Y)$ is uniformly convex.*

A proof of the first of these results may be found in [**2**, p. 98] while the second is proved in [**10**]. A useful consequence of the first theorem is

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