ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 1, Winter 1989

CONVEXITY PRESERVING CONVOLUTION OPERATORS

T.N.T. GOODMAN AND S.L. LEE

Dedicated to Professor A. Sharma

1. Introduction. Let K be a piecewise smooth, real 2π-periodic non negative function and $\gamma(t) = (f_1(t), f_2(t))$, $t \in [0, 2\pi]$, where $f_i(t)$ $(i = 1, 2)$ are piecewise smooth 2π-periodic functions, be a closed curve in $\mathbf{R}^2$. Then the function

$$(1.1) \qquad \Gamma(x) := \int_0^{2\pi} K(x-t)\gamma(t)dt, \quad x \in [0, 2\pi],$$

is also a closed curve in $\mathbf{R}^2$. The following theorem attributed to Loewner was proved in [**3**].

THEOREM (LOEWNER). *A necessary condition for the convolution transform* (1.1) *to map a positively convex curve onto a positively locally convex curve is that the closed curve* $(K'(x), K(x))$, $0 \leq x \leq 2\pi$, *is positively convex.*

The objectives of this paper are to show that the condition that the curve (K', K) be positively convex is also sufficient for the transformation (1.1) to map positively convex curves onto positively locally convex curves, and to give other sufficient conditions for the transformation to map convex curves onto convex curves. In §3, we mention an application to convolution operators with spline kernels from which the present work originates.

2. Convexity preserving kernels. As in §1, γ denotes a closed curve in $\mathbf{R}^2$. We shall assume that all curves are piecewise smooth. By *convexity* of γ we mean that it does not intersect any straight line more than twice. The kernel K in (1.1) is said to be *convex preserving*

Received by the editors on September 16, 1986.