

FULLY INVARIANT SUBMODULES OF p -LOCAL BALANCED PROJECTIVE GROUPS

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ABSTRACT. In this note we answer completely the question concerning the structure of fully invariant submodules of p -local balanced projective groups. In fact, every such submodule turns out to be a direct sum of an S -group and a balanced projective, and so its structure is completely determined by the well-known theories surrounding the S -groups and balanced projectives.

Ever since Warfield's initial work on balanced projective groups [4], there has been an open question concerning the structure of fully invariant subgroups of those groups. In this note we will provide a complete answer in the p -local case. We are able to show that every fully invariant submodule of a p -local balanced projective group is an SKT module (as introduced by Wick [5].) Some nice consequences of this result are that fully invariant submodules of p -local balanced projective groups are classified by a complete set of isomorphism invariants and that they satisfy general structural properties known for the class of isotype submodules of p -local balanced projective groups such as transitivity, full transitivity, and the equivalence of p^α -high submodules [2].

We will assume that all groups in this note are p -local and abelian; that is modules over the ring $\mathbf{Z}_p = \{\frac{m}{n} : m, n \in \mathbf{Z}, \text{ the ring of integers, with } (n, p) = 1\}$. Fully invariant submodules of a p -local group G are simply those submodules which contain their image under any endomorphism of G . It will be necessary to highlight the properties balanced projective p -local groups share which will be especially fruitful in our present study. Recall that the *height* of an element $x \in G$ is the ordinal α if $x \in p^\alpha G / p^{\alpha+1} G$, and x has height ∞ if $x \in p^\alpha G$ for all ordinals α . We will write $|x|_G$ to denote the height of x computed in G , and we will suppress the index when G is understood. An exact

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