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CONVERGENCE OF A CLASS OF DEFICIENT INTERPOLATORY SPLINES

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1. Introduction. The most popular choice for reasonably efficient approximating functions still continues to be in favor of cubic splines (see de Boor [**3**, p. 49]). The interpolation problems of matching a cubic spline at one intermediate point and a deficient cubic spline at two intermediate points between the successive mesh points have been studied by Meir and Sharma [**6**]. For this case, further studies in the direction of the results proved in [**6**] have been made in [**1**, **4** and **5**]. Instead of supplicating the additional degrees of freedom by prescribing values at two intermediate points for deficient cubic splines, our object is to study deficient cubic splines having two interpolatory conditions, one of which is the matching condition at intermediate points of the dividing intervals while the other is matching of derivatives at intermediate points. Interesting studies for sharp convergence properties for such spline interpolants when $f \in C^3$ or $f \in C^4$ have been made in the last section of the paper. It may be mentioned here that analysis to obtain the error bounds for interpolating deficient complex cubic splines is given by Chien-Kel Lu in [**2**].

2. Existence and uniqueness. Let a mesh on $[0,1]$ be given by

$$P : 0 = x_0 < x_1 < \cdots < x_n = 1$$

with $p = x_i - x_{i-1}$ for $i = 1, 2, \ldots, n$. For a positive integer m, π_m denotes the set of algebraic polynomials of degree not greater than m. For a function s defined over P, we denote the restriction of s over $[x_{i-1}, x_i]$ by s_i. The class of deficient cubic splines defined over P is given by

$$S(3,P) = \{s : s \in C^1[0,1], s_i \in \pi_3 \ \text{ for } i = 1, 2, \ldots, n\}.$$

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