ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 4, Summer 1988

A NOTE ON THE APPLICATION OF TOPOLOGICAL TRANSVERSALITY TO NONLINEAR DIFFERENTIAL EQUATIONS IN HILBERT SPACES

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ABSTRACT. In this paper we suggest a new method, via Topological Transversality, for examining nonlinear differential equations in Hilbert Spaces. Furthermore, we show how this analysis can be used to obtain existence of solutions to certain integro-differential equations.

1. Introduction. The theory of nonlinear differential equations in abstract spaces became popular in the 1970's and is still being studied in great depth. For a detailed account of the subject see Deimling [**4**], Lakshmikantham and Leela [**12**] and Martin [**14**]. In this paper we present a new approach via the Topological Transversality Theorem, to studying problems of the form

$$\begin{cases} y' = f(t,y), \ t \in [0,T] \\ y(0) = y_0. \end{cases} \tag{1.1}$$

Here y takes values in a real Hilbert space $(H, ||\cdot||)$, $y_0 \in H$ and $f : [0,T] \times H \to H$ is continuous.

For notational purposes let $C^1([0,T],H)$ denote the space of continuously differentiable functions g on $[0,T]$. Now $C^1([0,T],H)$ with norm

$$\begin{aligned} ||g||_1 &= \max\left\{ \sup_{t\in[0,T]} ||y(t)||, \sup_{t\in[0,T]} ||y'(t)|| \right\} \\ &= \max\left\{ ||y||_0, ||y'||_0 \right\} \end{aligned}$$

is a Banach space. Similarly we define $C([0,T],H)$. Finally, by a solution to (1.1) we mean a function $y \in C^1([0,T],H)$ together with y satisfying $y' = f(t,y), t \in [0,T]$, and $y(0) = y_0$.

Unlike the finite dimensional case, continuity assumptions on f alone will not guarantee even local existence; see Banas and Geobel [**2**]. In

Received by the editors on April 2, 1986 and in revised form on October 13, 1986.