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ON SOCLES OF ABELIAN P-GROUPS IN L

MANFRED DUGAS AND RENÉ VERGOHSEN

0. Introduction. All groups in this paper are (separable) abelian p-groups. Our notations are standard as in [**5**]. For set theoretic notations we refer to [**2**] or [**8**].

One of the most celebrated results in the theory of p-groups is Ulm's Theorem: Each *countable* p-group A is uniquely determined by its socle $A[p] = \{x \in A | px = 0\}$, viewed as a valuated Z/pZ-vectorspace with values induced by the height-function of A.

Since each countable, separable p-group is Σ-cyclic (i.e., a direct sum of cyclics), Ulm's Theorem doesn't provide much information in the case of separable p-groups. The Σ-cyclic and the torion-complete p-groups are the only ones known to be determined by their socles in the class of all separable p-groups. If we only want to deal with separable p-groups of *cardinality* $\aleph_1$, a result due to Hill and Megibben [**6**] reads as follows:

Assume $2^{\aleph_0} < 2^{\aleph_1}$. If A is neither Σ-cyclic nor torion-complete and A has a *countable* basic subgroup, then there exists a group A' such that A and A' are not isomorphic but the socles $A[p]$ and $A'[p]$ are isometric, i.e., there exists a height-preserving isomorphism $\sigma : A[p] \to A'[p]$.

Assuming that a consequence of Gödel's axiom of constructibility holds, namely, $\diamondsuit$(E) for each stationary subset E of $\aleph_1$, we will show that one may drop the countability condition in the Hill-Megibben theorem:

THEOREM ($V = L$). *Let A be a separable, abelian p-group of cardinality $\aleph_1$. If A is neither Σ-cyclic nor torion-complete, then there exists a p-group A' such that $A \cong A'$ but $A[p]$ and $A'[p]$ are isometric.*

In our second chapter, we study (weakly) ω_1-separable p-groups A of cardinality $\aleph_1$, cf. [**9**]. Such a group has an ω_1-filtration $A = \cup_{\nu<\omega_1} A_\nu$ into *pure*, countable subgroups A_ν such that $A_{\nu+1}$ is a summand of