ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 4, Summer 1988

WEAKLY CLOSE-TO-CONVEX FUNCTIONS

ALBERT E. LIVINGSTON

ABSTRACT. We obtain coefficient bounds and integral means inequalities for the class of multivalent weakly close to convex functions. We also consider the integral means problem for the subclass of multivalent convex functions.

Introduction. Let $S(p)$ denote the class of functions f, analytic in $\Delta = \{z : |z| < 1\}$, with p zeros, counting multiplicity, in Δ and such that there exists $\delta > 0$ so that $\mathrm{Re}\,[zf'(z)/f(z)] > 0$ for $\delta < |z| < 1$. Functions in $S(p)$ are called p-valent starlike functions. Hummel [**11**] extended $S(p)$ to the class of weakly starlike functions and this was further extended by Styer [**21**]. Following Styer, we say that a function is a member of $S_{wc}(p)$ if and only if there exists a sequence of functions f_n in $S(p)$ such that f_n converges to f uniformly on compact subsets of Δ. Equivalently [**21**], f is in $S_{wc}(p)$ if and only if there exists h in $S(1)$, with $h(0) = h'(0) - 1 = 0$, such that

$$f(z) = [h(z)]^p \prod_{j=1}^{p} \frac{(z-\alpha_j)(1-\overline{\alpha}_j z)}{z}, \quad |\alpha_j| \leq 1. \tag{1.1}$$

The fundamental difference between Styer's definition and Hummel's is that Hummel's requires $|\alpha_j| < 1$.

The author [**13**] studied the class of close to convex functions $K(p)$. A function F, analytic in Δ with $F(0) = 0$, is in $K(p)$ if and only if there exists f in $S(p)$ with $f(0) = 0$ and a $\delta > 0$ such that $\mathrm{Re}\,[zF'(z)/f(z)] > 0$ for $\delta < |z| < 1$. Styer [**22**] extended this class to the class of weakly close to convex functions $K_w(p)$, giving several equivalent characterizations of $K_w(p)$. A function F, not identically zero in Δ, is in $K_w(p)$ if and only if there is a sequence of functions F_n in $K(p)$ such that F_n converges to F uniformly on compact subsets of Δ. Equivalently F is in $K_w(p)$ if and only if $F(0) = 0$ and there is a function f in $S_{wc}(p)$ with $f(0) = 0$, such that $\mathrm{Re}\,[zF'(z)/f(z)] > 0$ in

Received by the editors on March 18, 1986, and in revised form on July 25, 1986.